Against God of the Truth-Value Gaps

T. Parent

1. Introduction

Can God create a stone that God cannot lift? The paradox implicit here can be formulated as follows:\(^1\)

(P1) Either God can create an unliftable stone, or not.

(P2) If so, then there is a possible task that God cannot do (viz., lift the stone).

(P3) If not, then there is a possible task that God cannot do (viz., create the stone).

(C) So, either way, there is a possible task which God cannot do.

The conclusion is seen as refuting that God is omnipotent. This of course is at odds with traditional theism.

Mavrodes (1963), following Aquinas (1947, I.25.3), resolves this by analysing ‘an unliftable stone’ into the inconsistent expression ‘a stone that cannot be lifted by a being who can lift any stone’. Concurrently, (P3) is regarded as false; God creating such a stone would not be a possible task. The necessary non-existence of the stone then does not reflect a limit

---

\(^1\) The present formulation of the paradox is not quite the traditional formulation. That formulation suggests *inter alia* that lifting the stone is something which God could not do, and so, God cannot do everything. But as Swinburne (1973) explains, this is technically a non-sequitur. For God might still have the power to perform any actual task (and so actually be omnipotent), even if there are possible tasks that God cannot perform. Yet as Swinburne also explains, this offers no solace to traditional theism, which holds that God is omnipotent only if God can do any possible task. Accordingly, the above formulation is meant to get to the essential issue with fewer epicycles.
God’s power but rather just the logical impossibility of the object.2 Not every theist may be content with regarding God as bound by laws of logic; however, the solution from Aquinas has been influential enough to deserve the label the ‘orthodox’ solution.3

Recently, however, a novel solution has been offered by Beall & Cotnoir (2017) (henceforth, ‘B&C’). The idea is to regard (P1) as gappy in truth-value – as neither true nor false – along with certain other instances of the Law of Excluded Middle (LEM) concerning God’s power. Thus, B&C claim that ‘one can no more falsely attribute limits to God’s omnipotence than one can truly do so’ (p. 682, italics omitted). In what follows, however, I shall explain that the gappy solution either fails or is parasitic on Aquinas’ solution. And in the latter case, it is better just to prefer Aquinas’ solution, rather than burden it with the further claim that there are truth-value gaps regarding God’s power. Accordingly, I conclude that B&C’s proposal offers no new promising avenues for traditional theism.

2. Outline of B&C’s proposal

Let me first say, however, that B&C’s view has many attractive features. The logic K3 (reviewed in B&C’s appendix) offers a rigorous way to flesh out their proposal – and since there are independent doubts about LEM, it would be unfair to dismiss B&C’s solution as ad hoc. Quite the contrary, one might expect LEM to fail in discussing something as abstruse as God’s infinite power. More generally, I strongly concur with B&C that ‘[t]he

---

2 Per usual in these discussions, talk of what one ‘can do’ is interpreted as equivalent to talk of what one has the ‘power’ to do. A consequence is that if $S$ actually $\phi$s, then $S$ can $\phi$—meaning $S$ has the ‘power’ to $\phi$. There may be interpretations of the word ‘power’ where this does not hold up, but I assume that there is at least one (relevant) interpretation where it does. My thanks to Reviewer #1 for this journal in encouraging me to clarify this.

3 There are also solutions that resist (P2); see Frankfurt (1964) and Swinburne (1994).
marriage of philosophically familiar nonclassical logics and philosophical theology is long overdue’ (p. 687). Though again, I do not believe that B&C’s specific marriage proposal is ultimately desirable.

To repeat, B&C’s central claim is that some statements concerning God’s powers flout LEM. And yet, B&C think that many truths of the form \( p \lor \neg p \) still hold, even when it comes to talk of God’s powers. For example, they need not reject LEM-instances such as ‘either God is omnipotent or not,’ ‘either God can exist eternally or not,’ etc. It is therefore important to identify which instances of LEM B&C end up rejecting. In this connection, B&C offer the following claim (G) about ‘limit claims,’ i.e., claims concerning limits on God’s powers (quoting p. 684):

\[(G) \text{ If a limit claim } L \text{ implies that God has limits, and the negation of } L \text{ implies that God has limits, then } L \text{ is gappy. Otherwise, } L \text{ is false.}\]

In the stone paradox, the relevant implication of (G) would be:

\[(G1) \text{ If ‘God can create an unliftable stone’ implies that God has limits, and its negation also implies that God has limits, then ‘God can create an unliftable stone’ is gappy.}\]

Observe that (G1), (P2), and (P3) entail that ‘God can create an unliftable stone’ is neither true nor false: If it is true, then (P2) secures that God has limits; if it is false, then (P3) secures it instead – so given (G1), the sentence is gappy. In K3, moreover, the negation of a gappy statement is itself gappy – ditto for a disjunction formed from a gappy statement and its negation. It thus follows that premise (P1) in the paradox also has a truth-value gap.

As B&C are aware, it may sound strange that an omnipotent being does not have the power to create the stone. Indeed, B&C grant that on one reading of ‘omnipotent’, that is indeed unacceptable. But B&C propose an alternative reading where God qualifies as ‘omnipotent’ as long as God does not lack any power. And so, given that God neither has nor
lacks the power to create the stone, the suggestion is that in a perfectly legitimate sense, God remains omnipotent. (I suspect this will not satisfy some traditional theists; regardless, I shall not quarrel with the matter below.)

3. A revenge paradox

Again, B&C propose that certain sentences concerning God’s power are truth-value gaps. But the basic question I want to ask is: Can B&C’s God make such sentences non-gappy? More precisely, can God avoid having gappy power-ascriptions (gappy ‘PAs’)? Regardless of the answer here, we can show that B&C’s God must have limits.

For this purpose, let us first establish the following lemma:

(L1) If God must have at least one gappy PA, then God’s power has limits.

Argument for (L1): Consider that no ascription of a power to me violates LEM. Hence, I am capable of having no gappy PAs. Yet anything that I can do is something B&C’s God can do – or at least, their God does not lack the power of doing it.4 However, the antecedent of (L1) implies that God positively cannot avoid a gappy PA. So, for B&C’s God, the antecedent of (L1) is false, meaning (L1) is true.

But as a second lemma, we can also show that if B&C’s God can avoid having gappy PAs, then their God is also limited. The argument here is as follows:

(1) If God can avoid gappy PAs, then there is a possible world W where God has no gappy PAs.

(2) If there is a possible world W where God has no gappy PAs, then at W: Either

God can create an unliftable stone, or not.

4 Thanks to Reviewer #2 for this journal for emphasizing the latter disjunct to me—this caused me to reformulate significantly the main argument of this paper.
(3) If at W: Either God can create an unliftable stone, or not, then at W: There is a possible task which God cannot do.

(4) If at W: There is a possible task that God cannot do, then there is a possible task that God cannot do.

(5) If there is a possible task that God cannot do, then God’s power has limits.

(L2) So, if God can avoid gappy PAs, then God’s power has limits. [From (1) – (5)]

Thus, if (L1) suggests that an omnipotent God can avoid gappy PAs, (L2) then indicates that God still has limits.

Let us review premises (1) – (5) in reverse order. First, (5) seems assumed by traditional theism (cf. note 1). Second, (4) is correct if modal accessibility is an equivalence relation, as seen in a modal system such as S5. (This is an approach where all possible worlds can ‘see’ all possible worlds.) B&C could thus resist the argument by rejecting S5; however, that would be a contentious stance which, I assume, one would hope to avoid.5

As for (3), this can be justified by readily adapting the initial argument at (P1) – (C). We need only modify the argument by inserting the operator ‘at W’ where appropriate, e.g., at the start of (P1). The modified argument then effectively justifies the consequent of (3) from the antecedent of (3).

Premise (2) is true by the definition of ‘gappy PA’, whereas premise (1) I regard as fairly uncontroversial, at least in the present context. The existence of possible worlds is of course controversial in other contexts, but the ontology of such worlds is not the issue here.

5 Reviewer #1 asks whether the argument shows just that God’s power is limited in some world. It indeed shows at least that; however, axiom S5 also ensures that a possibly limited God is an actually limited God as well. N.B., there are philosophers who reject S5 on independent grounds, e.g., Salmon (1989). Yet if rejecting S5 is required by B&C’s solution, then that may make their solution compare unfavourably to, e.g., the orthodox solution.
The premise is just using the possible-worlds vocabulary to express that if God has a certain kind of power, then such-and-such is possible.

4. Another gap?

The reader may have noticed, however, that the antecedents of (L1) and (L2) form a disjunction along the lines of:

(*) Either God must have a gappy PA, or not.

And the lemmas tell us that either disjunct implies limits on God. That, moreover, is what lead to the suggestion that B&C are vulnerable to a ‘revenge paradox.’ However, this is just the sort of situation which initially lead B&C to hypothesize that (P1) is a truth-value gap. So, in the same way, (G) might be invoked here to suggest that (*) is also a gap. Apparently, both ‘God must have a gappy PA’ and its negation each imply that God has limits. Thus, (G) implies that both it and its negation are gappy, meaning that (*) is gappy. And that would be enough to thwart the revenge reasoning suggesting that B&C’s God has limits.

In reply, I will argue for the following:

(†) If (*) is true at any world V, then God’s power has limits.

If this is correct, then principle (G) will imply that in every world, (*) is a truth-value gap. By the semantics for K3, that means its second disjunct is gappy in every world. This, in turn, indicates that God necessarily has a gappy ‘meta-PA’. That is to say, the following PA about PAs will be gappy of necessity: ‘God can avoid having gappy PAs’.

The crux, however, is that the gappy meta-PA is itself a gappy PA. It is a gappy ascription of the power to have no gappy PAs. Thus, if God has the gappy meta-PA in every world, then God must have at least one gappy PA. That, moreover, contradicts the idea that (*) lacks a truth-value. For it reveals that the first disjunct of (*) is true. In which case, assigning (*) a truth-value gap is not a workable strategy for B&C.
To complete our case, then, we need only show that (†) is true. The argument in outline is that in an arbitrary world \( V \), either disjunct of (*) leads to God’s power having limits. *A propos* the first disjunct, we begin by showing:

(L3) If at \( V \): God must have a gappy PA, then God’s power has limits.

Argument for (L3): If God must have a gappy PA at \( V \), then at \( V \), there is a possible task that God cannot do, viz., avoid gappy PAs.⁶ More, if at \( V \) there is a possible task which God cannot do, then by axiom S5, there is a possible task which God cannot do. And by (5), this means God’s power has limits. Therefore, if God must have a gappy PA at \( V \), then God’s power has limits, which is what (L3) says.

The case for (†) also requires that the second disjunct of (*) in an arbitrary \( V \) implies that God’s power has limits. The argument for this fourth lemma (in brief) is that \((1) – (L2)\) is a piece of sound reasoning not just in the actual world, but in any world \( V \). In more detail, it is just as well for that reasoning to begin with the following:

\((1_\nu)\) If at \( V \): God can avoid gappy PAs, then there is a possible world \( W \) where God has no gappy PAs.

Note also that (2) and (3) remain correct. By two uses of hypothetical syllogism, then, \((1_\nu), (2),\) and (3) entail:

\((3_\nu)\) If at \( V \): God can avoid gappy PAs, then at \( W \): There is a possible task that God cannot do.

Further, axiom S5 still vindicates (4). So, assuming (5), \((3_\nu)\) and (4) establish that:

(L4) If at \( V \): God can avoid gappy PAs, then God’s power has limits.

⁶ ‘Avoiding gappy PAs’ is by far the norm for individuals that we are acquainted with. So, it is assumed that in any world \( V \), having no gappy PAs is at least possible. As with axiom S5, if B&C deny such an assumption, that would seem to be a liability. There remains some question, however, of whether it is possible for God to have no gappy PAs. I address this at the end of the present section.
Thus, if the second disjunct of (*) holds at some world V, then God’s power has limits.

Taken together, (L3) and (L4) indicate that if (*) is not gappy at V, then God’s power has limits. This is what (†) says. Again, the significance of (†) is that necessarily B&C’s God has the gappy meta-PA ‘God can avoid gappy PAs’. That suffices for God to have a gappy PA in every world. Yet that is exactly what the first disjunct of (*) says. Thus (*) is true, not a truth-value gap – so it is not open to B&C to assign (*) a gap.

In reply to all this, B&C may attempt to reject (L3) by suggesting that it is just flatly impossible for God to avoid gappy PAs, much like it is impossible to square a circle. And it need not impugn God’s omnipotence that God cannot realize an impossibility. Yet that was the sort of idea that allowed Aquinas to elude the stone paradox in the first place, without appeal to truth-value gaps. So, if B&C take this stance, they seem to fall back on Aquinas’ solution – in which case, there is little motivation to adopt their gappy solution besides.7

5. Closing

The conclusion is that B&C’s solution is threatened by a revenge paradox. We have seen that there is one way for them to avoid it; however, the cost is that they end up depending on Aquinas’ solution. And since that solution is enough to stall the original paradox, there becomes little reason for B&C to proffer truth-value gaps in addition. Apparently, then, B&C’s solution to the paradox is either inadequate or not well-motivated.8

---

7 Relatedly, B&C might say that God can avoid gappy PAs per se, but that God cannot both avoid gappy PAs and be capable of creating the stone. That would allow B&C to stymie the argument for (3, ), besides instituting a surface-level difference with Aquinas. However, the root rationale would seem to be the Aquinian one—that some situations are just flatly impossible, and that God’s inability to realize them does not flout God’s omnipotence. Here too, B&C’s solution would just become an overburdened version of Aquinas’.

8 I thank Jim Hutchinson and two anonymous reviewers for excellent feedback on previous drafts.
References


nontology@gmail.com

Dept. of History, Philosophy, and Religious Studies

Nazarbayev University

Republic of Kazakhstan