Chapter 2

SENTENTIAL LOGIC: SYMBOLIZATION AND SYNTAX

2.1 SYMBOLIZATION AND TRUTH-FUNCTIONAL CONNECTIVES

Sentential logic, as the name suggests, is a branch of formal logic in which sentences are the basic units. In this chapter we shall introduce *SL*, a symbolic language for sentential logic, which will facilitate our development of formal techniques for assessing the logical relations among sentences and groups of sentences. The sentences of English that can be symbolized in *SL* are those that are either true or false, that is, have truth-values.

In English there are various ways of generating sentences from other sentences. One way is to place a linking term such as 'and' between them. The result, allowing for appropriate adjustments in capitalization and punctuation, will itself be a sentence of English. In this way we can generate

Socrates is wise and Aristotle is crafty

by writing 'and' between 'Socrates is wise' and 'Aristotle is crafty'. Some other linking terms of English are 'or', 'although', 'unless', 'before', and 'if and only if'. As used to generate sentences from other sentences, these terms are called

sentential connectives (they connect or join sentences to produce further sentences).

Some sentence-generating words and expressions do not join two sentences together but rather work on a single sentence. Examples are 'it is not the case that' and 'it is alleged that'. Prefacing a sentence with either of these expressions generates a further sentence. Since these expressions do not literally connect two sentences, the term "sentential connective" is perhaps a little misleading. Nonetheless, such sentence-generating devices as these are commonly classified as sentential connectives, and we shall follow this usage.

Sentences generated from other sentences by means of sentential connectives are **compound sentences.** All other sentences are **simple sentences.** In developing sentential logic we shall be especially interested in the **truth-functional** use of sentential connectives. Intuitively a compound sentence generated by a truth-functional connective is one in which the truth-value of the compound is a function of, or is fixed by, the truth-values of its components.

> A sentential connective is used *truth-functionally* if and only if it is used to generate a compound sentence from one or more sentences in such a way that the truth-value of the generated compound is wholly determined by the truth-values of those one or more sentences from which the compound is generated, no matter what those truth-values may be.

Few, if any, connectives of English are always used truth-functionally. However, many connectives of English are often so used. We shall call these connectives, as so used, **truth-functional connectives.** A **truth-functionally compound sentence** is a compound sentence generated by a truth-functional connective.

In English 'and' is often used truth-functionally. Consider the compound sentence

Alice is in England and Bertram is in France.

Suppose that Alice is in Belgium, not England. Then 'Alice is in England' is false. The compound sentence is then clearly also false. Similarly, if 'Bertram is in France' is false, the compound 'Alice is in England and Bertram is in France' is false as well. In fact, this compound will be true if and only if both of the sentences from which it is generated are true. Hence the truth-value of this compound is wholly determined by the truth-values of the component sentences from which it is generated. Given their truth-values, whatever they may be, we can always compute the truth-value of the compound in question. This is just what we mean when we say that 'and' functions as a truth-functional connective.

SENTENCES OF SENTENTIAL LOGIC

In *SL* capital Roman letters are used to abbreviate individual sentences of English. Thus

Socrates is wise

can be abbreviated as

W

Of course, we could have chosen any capital letter for the abbreviation, but it is common practice to select a letter that reminds us of the sentence being abbreviated. In this case 'W' reminds us of the word 'wise'. But it is essential to remember that the capital letters of *SL* abbreviate entire sentences and *not* individual words within sentences.

To ensure that we have enough sentences in our symbolic language to represent any number of English sentences, we shall also count capital Roman letters with positive-integer subscripts as sentences of *SL*. Thus all the following are sentences of *SL*:

A, B, Z, T_{25}, Q_6

In *SL* capital letters with or without subscripts are **atomic sentences.** Sentences of *SL* that are made up of one or more atomic sentences and one or more sentential connectives of *SL* are *molecular sentences.*

CONJUNCTION

We could abbreviate

Socrates is wise and Aristotle is crafty

in our symbolic language as 'A', but in doing so we would bury important information about this English sentence. This sentence is a compound made up of two simple sentences: 'Socrates is wise' and 'Aristotle is crafty'. Furthermore, in this case the word 'and', which connects the two sentences, is serving as a truth-functional connective. This compound sentence is true if both of its component sentences are true and is false otherwise. We shall use '&' (ampersand) as the sentential connective of *SL* that captures the force of this truth-functional use of 'and' in English. Instead of symbolizing 'Socrates is wise and Aristotle is crafty' as 'A', we can now symbolize it as

W & C

where 'W' abbreviates 'Socrates is wise' and 'C' abbreviates 'Aristotle is crafty'. Remember that the letters abbreviate entire sentences, not merely specific words like the words 'wise' and 'crafty'. The compound sentence 'W & C' is an example of a molecular sentence of *SL*.

A sentence of the form

P & **Q**

where **P** and **Q** are sentences of *SL*, is a **conjunction.**[1] **P** and **Q** are the **conjuncts** of the conjunction. Informally we shall use the terms "conjunction" and "conjunct" in talking of English sentences that can be symbolized as conjunctions of *SL*. The relation between the truth or falsity of a conjunction and the truth or falsity of its conjuncts can be simply put: A conjunction is true if and only if both of its conjuncts are true. This is summarized by the following table:

P	**Q**	**P** & **Q**
T	T	T
T	F	F
F	T	F
F	F	F

Such a table is called a *characteristic truth-table* because it defines the use of '&' in *SL*. The table is read horizontally, row by row. The first row contains three **T**'s. The first two indicate that we are considering the case in which **P** has the truth-value **T** and **Q** has the truth-value **T**. The last item in the first row is a **T**, indicating that the conjunction has the truth-value **T** under these conditions. The second row indicates that, when **P** has the truth-value **T** and **Q** has the truth-value **F**, the conjunction has the truth-value **F**. The third row shows that, when **P** has the truth-value **F** and **Q** has the truth-value **T**, the conjunction has the truth-value **F**. The last row indicates that when both **P** and **Q** have the truth-value **F**, the conjunction has the truth-value **F** as well.

Sometimes an English sentence that is not itself a compound sentence can be paraphrased as a compound sentence. The sentence

Fred and Nancy passed their driving examinations

can be paraphrased as

Both Fred passed his driving examination and Nancy passed her driving examination.

We underscore the connectives in paraphrases to emphasize that we are using those connectives truth-functionally. We use 'both . . . and . . .',

[1] Our use of boldface letters to talk generally about the sentences of *SL* is explained in Section 2.4.

rather than just 'and', to mark off the conjuncts unambiguously. Where the example being paraphrased is complex, we shall sometimes also use parentheses—'(' and ')'—and brackets—'[' and ']'—to indicate grouping. The foregoing paraphrase is an adequate paraphrase of the original sentence inasmuch as both the original sentence and the paraphrase are true if and only if 'Fred passed his driving examination' and 'Nancy passed her driving examination' are both true. The paraphrase is a conjunction and can be symbolized as

F & N

where 'F' abbreviates 'Fred passed his driving examination' and 'N' abbreviates 'Nancy passed her driving examination'.

Symbolizing English sentences in *SL* should be thought of as a two-step process. First, we construct in English a truth-functional paraphrase of the original English sentence; next, we symbolize that paraphrase in *SL*. The paraphrase stage serves to remind us that the compounds symbolized as molecular sentences of *SL* are always truth-functional compounds.

The preceding example illustrates that the grammatical structure of an English sentence is not a completely reliable indication of its logical structure. Key words like 'and' serve as clues but are not infallible guides to symbolization. The sentence

Two jiggers of gin and a few drops of dry vermouth make a great martini

cannot be fairly paraphrased as

Both two jiggers of gin make a great martini and a few drops of dry vermouth make a great martini.

Together these ingredients may make a great martini, but separately they make no martini at all. Such a paraphrase completely distorts the sense of the original sentence. Thus the original sentence must be regarded as a simple sentence and symbolized in *SL* as an atomic sentence, say

M

Many sentences generated by such other connectives of English as 'but', 'however', 'although', 'nevertheless', 'nonetheless', and 'moreover' can be closely paraphrased using 'and' in its truth-functional sense. Consider some examples:

Susan loves country music, but she hates opera

can be paraphrased as

Both Susan loves country music and Susan hates opera.

The paraphrase can be symbolized as 'L & H', where 'L' abbreviates 'Susan loves country music' and 'H' abbreviates 'Susan hates opera'.

The members came today; however, the meeting is tomorrow

can be paraphrased as

Both the members came today and the meeting is tomorrow

which can be symbolized as 'C & M', where 'C' abbreviates 'The members came today' and 'M' abbreviates 'The meeting is tomorrow'.

Although George purchased a thousand raffle tickets, he lost

can be paraphrased as

Both George purchased a thousand raffle tickets and George lost

which can be symbolized as 'P & L', where 'P' abbreviates 'George purchased a thousand raffle tickets' and 'L' abbreviates 'George lost'.

In each of these cases, the paraphrase perhaps misses part of the sense of the original English sentence. In the last example, for instance, there is the suggestion that it is surprising that George could have purchased a thousand raffle tickets and still have lost the raffle. Truth-functional paraphrases often fail to capture all the nuances present in the sentences of which they are paraphrases. This loss is usually not important for the purposes of logical analysis.

In symbolizing sentences of a natural language—in our case English—grammatical structure and key words provide important clues, but they are not infallible guides to correct symbolizations. Ultimately we have to ask ourselves, as speakers of English, whether the sentence can be reasonably paraphrased as a truth-functional compound. If so, we can symbolize it as a molecular sentence of *SL*. If not, we have to symbolize it as an atomic sentence of *SL*.

DISJUNCTION

Another sentential connective of English is 'or', used in such sentences as

Henry James was a psychologist or William James was a psychologist.

This English sentence contains two simple sentences as components: 'Henry James was a psychologist' and 'William James was a psychologist'. The truth-value of the compound wholly depends upon the truth-values of the component sentences. As long as at least one of the component sentences is true, the compound is true; but if both the components are false, then the compound is false. When used in this way, 'or' serves as a truth-functional connective of English. In *SL* '∨' (wedge) is the symbol that expresses this truth-functional relation. Thus the sentence about Henry and William James can be symbolized as

H ∨ W

where 'H' abbreviates 'Henry James was a psychologist' and 'W' abbreviates 'William James was a psychologist'. 'H ∨ W' is true if 'H' is true or 'W' is true, and it is false only when both 'H' and 'W' are false.

A sentence of the form

P ∨ Q

where **P** and **Q** are sentences of *SL*, is a **disjunction. P** and **Q** are the **disjuncts** of the sentence. Informally we shall use the terms "disjunction" and "disjunct" in talking of English sentences that can be symbolized as disjunctions of *SL*. A disjunction is true if and only if at least one of its disjuncts is true. This is summarized by the following characteristic truth-table:

P	**Q**	**P ∨ Q**
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

The only case in which a disjunction has the truth-value **F** is when both disjuncts have the truth-value **F**.

Some sentences of English that do not contain the word 'or' can be paraphrased as a disjunction. For instance,

At least one of the two hikers, Jerry and Amy, will get to the top of the mountain

can adequately be paraphrased as

<u>Either</u> Jerry will get to the top of the mountain <u>or</u> Amy will get to the top of the mountain.

This paraphrase can be symbolized as 'J ∨ A', where 'J' abbreviates 'Jerry will get to the top of the mountain' and 'A' abbreviates 'Amy will get to the top of the mountain'. Remember, the letters abbreviate the entire sentences, not just the words 'Jerry' and 'Amy'. In paraphrasing English sentences as disjunctions of *SL*, we use the '<u>either</u> . . . <u>or</u> . . .' construction to mark off the two disjuncts unambiguously.

In English sentences that can be paraphrased as disjunctions, 'or' does not always occur between full sentences. For example,

> Nietzsche is either a philosopher or a mathematician

can be paraphrased as

> Either Nietzsche is a philosopher or Nietzsche is a mathematician.

This truth-functional paraphrase can be symbolized as 'P ∨ M', where 'P' abbreviates 'Nietzsche is a philosopher' and 'M' abbreviates 'Nietzsche is a mathematician'.

We use the wedge to symbolize disjunctions in the *inclusive* sense. Suppose the following appears on a menu:

> With your meal you get apple pie or chocolate cake.

We might try to paraphrase this as

> Either with your meal you get apple pie or with your meal you get chocolate cake.

Since we use 'or' only in the inclusive sense in paraphrases, this paraphrase is true if either or both of the disjuncts are true. In ordinary English, on the other hand, 'or' is sometimes used in a more restrictive sense. In the present example, if someone orders both pie and cake, the waiter is likely to point out that either cake or pie, but *not* both, comes with the dinner. This is the exclusive sense of 'or'—either one or the other but not both. Although this sense of 'or' cannot be captured by '∨' alone, there is, as we shall soon see, a combination of connectives of *SL* that will allow us to express the exclusive sense of 'or'.

NEGATION

'It is not the case that' is a sentential connective of English. Consider the following compound generated by this connective:

> It is not the case that Franklin Pierce was president.

This sentence is true if its component sentence, 'Franklin Pierce was president', is false, and it is false if that component sentence is true. 'It is not the case that' is a truth-functional connective because the truth-value of the generated sentence is wholly determined by the truth-value of the component sentence. In *SL* '~ ' (tilde) is the sentential connective that captures this truth-functional relationship. Thus the sentence in question can be symbolized as

~ F

where 'F' abbreviates 'Franklin Pierce was president'. The tilde is a **unary connective,** because it "connects" only one sentence. On the other hand, '&' and '∨' are **binary connectives** since each connects two sentences. When '~ ' is placed in front of a sentence, the truth-value of the generated sentence is the opposite of the truth-value of the original sentence. So the characteristic truth-table for negation is this:

P	~ P
T	F
F	T

Notice that, because '~ ' is a unary connective, we need a truth-table of only two rows to represent all the possible "combinations" of truth-values that a single sentence to which '~ ' is attached might have.

Putting a '~ ' in front of a sentence forms the negation of that sentence. Hence '~ A' is the negation of 'A' (though 'A' is *not* the negation of '~ A'), '~ ~ A' is the negation of '~ A' (though '~ A' is not the negation of '~ ~ A'), and so forth. Informally we shall use the term "negation" in talking about sentences of English that can be symbolized as negations in *SL*. Thus

It is not the case that Franklin Pierce was president

is the negation of

Franklin Pierce was president.

Whether an English sentence should be symbolized as a negation depends on the context. As before, grammar and key words give us clues. Consider some examples:

Not all sailors are good swimmers

is readily paraphrased as

It is not the case that all sailors are good swimmers.

This paraphrase can be symbolized as '~ G', where 'G' abbreviates 'All sailors are good swimmers'. But the following example is not as straightforward:

No doctors are rich.

One might be tempted to paraphrase this sentence as 'It is not the case that all doctors are rich', but to do so is to treat 'No doctors are rich' as the negation of 'All doctors are rich'. This is a mistake because a sentence and its negation are so related that, if one is true, the other is false, and vice versa. In fact, since some doctors are rich and some doctors are not rich, both 'All doctors are rich' and 'No doctors are rich' are false. Hence the latter cannot be the negation of the former. Rather, 'No doctors are rich' is the negation of 'Some doctors are rich'. 'No doctors are rich' is true if and only if 'Some doctors are rich' is false, so the former sentence can be paraphrased as

It is not the case that some doctors are rich.

This can be symbolized as '~ D', where 'D' abbreviates 'Some doctors are rich'.

Some further examples will be helpful:

Chlorine is not a metal

can plausibly be understood as

It is not the case that chlorine is a metal.

This paraphrase can be symbolized as '~ C', where 'C' abbreviates 'Chlorine is a metal'. Notice that 'Chlorine is a metal' and 'Chlorine is not a metal' are such that if either is true the other is false, which must be the case if the latter is to be the negation of the former. But now consider an apparently similar case:

Some humans are not male.

This sentence should not be paraphrased as 'It is not the case that some humans are male'. The latter sentence is true if and only if *no* humans are male, which is not the claim made by the original sentence. The proper paraphrase is

It is not the case that all humans are male

which can be symbolized as '~ H', where 'H' abbreviates 'All humans are male'. Often sentences containing words with such prefixes as 'un-', 'in-', and 'non-' are best paraphrased as negations. But we must be careful here.

Kant was unmarried

can be understood as

It is not the case that Kant was married and then symbolized as '~ K', where 'K' abbreviates 'Kant was married'. 'Kant was unmarried' is the negation of 'Kant was married'. But

Some people are unmarried

should not be paraphrased as 'It is not the case that some people are married'. 'Some people are married' and 'Some people are unmarried' are both true. A proper paraphrase in this case is

It is not the case that all people are married

which can be symbolized as '~ M', where 'M' abbreviates 'All people are married'.

COMBINATIONS OF SENTENTIAL CONNECTIVES

So far we have discussed three types of truth-functional compounds—conjunctions, disjunctions, and negations—and the corresponding sentential connectives of *SL*— '&', '∨', and '~ '. These connectives can be used in combination to symbolize complex passages. Suppose we wish to symbolize the following:

Either the steam engine or the computer was the greatest modern invention, but the zipper, although not the greatest modern invention, has made life much easier.

The main connective in this sentence is 'but', and the sentence can be paraphrased as a conjunction. The left conjunct can be paraphrased as a disjunction, and the right can be paraphrased as a conjunction making the claim that the zipper was not the greatest modern invention and the claim that the zipper has made life much easier. Finally the claim that the zipper was not the greatest modern invention can be paraphrased as a negation. The resulting truth-functional paraphrase is

Both (either the steam engine was the greatest modern invention or the computer was the greatest modern invention) and (both it is not the case that the zipper was the greatest modern invention and the zipper has made life much easier).

For clarity we have inserted some parentheses in the paraphrase to emphasize the grouping of the components. The order of placement of 'both' and 'either' is important. In this case 'both' occurring before 'either' at the beginning shows that the overall sentence is a conjunction, not a disjunction. The paraphrase can be symbolized as

(S ∨ C) & (~ Z & E)

where 'S' abbreviates 'The steam engine was the greatest modern invention', 'C' abbreviates 'The computer was the greatest modern invention', 'Z' abbreviates 'The zipper was the greatest modern invention', and 'E' abbreviates 'The zipper has made life much easier'.

The connectives '&', '∨', and '~ ' can be used in combination to symbolize English sentential connectives such as 'neither . . . nor . . .'. The sentence

Neither Sherlock Holmes nor Watson is fond of criminals

can be paraphrased as

Both it is not the case that Sherlock Holmes is fond of criminals and it is not the case that Watson is fond of criminals.

This can be symbolized as

~ H & ~ W

where 'H' abbreviates 'Sherlock Holmes is fond of criminals' and 'W' abbreviates 'Watson is fond of criminals'.

Another equally good paraphrase of the original sentence is

It is not the case that either Sherlock Holmes is fond of criminals or Watson is fond of criminals.

This paraphrase can be symbolized using the above abbreviations as

~ (H ∨ W)

Note that the original sentence, the paraphrases, and the symbolic sentences are all true if Sherlock Holmes is not fond of criminals and Watson is not fond of criminals, and they are all false otherwise.

A similar, but nonequivalent, connective is 'not both . . . and . . .'. Consider this claim:

A Republican and a Democrat will not both become president.

Truth-functionally paraphrased this becomes

It is not the case that both a Republican will become president and a Democrat will become president

which is symbolized as

~ (R & D)

This sentence does not maintain that neither a Republican nor a Democrat will become president but only that not both of them will become president. '~ (R ∨ D)' is not an acceptable symbolization, but '~ (R & D)' is. Another possible and acceptable paraphrase of this particular 'not both . . . and . . .' claim is

> Either it is not the case that a Republican will become president or it is not the case that a Democrat will become president

which when symbolized becomes

> ~ R ∨ ~ D

Here is a table summarizing the truth conditions for 'neither . . . nor . . .'. Notice that a 'neither . . . nor . . .' expression is true only when both of its components, **P** and **Q**, are false.

Truth Conditions for
'Neither . . . nor . . .'

P	**Q**	**~ P & ~ Q**	**~ (P ∨ Q)**
T	**T**	**F**	**F**
T	**F**	**F**	**F**
F	**T**	**F**	**F**
F	**F**	**T**	**T**

Compare this table with the next table, which shows the truth conditions for 'not both . . . and . . .':

Truth Conditions for
'Not both . . . and . . .'

P	**Q**	**~ (P & Q)**	**~ P ∨ ~ Q**
T	**T**	**F**	**F**
T	**F**	**T**	**T**
F	**T**	**T**	**T**
F	**F**	**T**	**T**

A 'not both . . . and . . .' expression is false only when both of its components, **P** and **Q**, are true.

A combination of the sentential connectives of *SL* can also be used to capture the exclusive sense of 'or' discussed earlier. Recall that the sentence

> With your meal you get apple pie or chocolate cake

is true in the exclusive sense of 'or' if with your meal you get apple pie or chocolate cake but not both apple pie and chocolate cake. We now know how to paraphrase the 'not both . . . and . . .' portion of the sentence. The paraphrase of the whole sentence is

> Both (either with your meal you get apple pie or with your meal you get chocolate cake) and it is not the case that (both with your meal you get apple pie and with your meal you get chocolate cake).

This can be symbolized as

$$(A \vee C) \ \& \sim (A \ \& \ C)$$

where 'A' abbreviates 'With your meal you get apple pie' and 'C' abbreviates 'With your meal you get chocolate cake'. Here is a table showing the truth conditions for exclusive 'or':

Truth Conditions for Exclusive 'Or'

P	Q	(P ∨ Q) & ~ (P & Q)
T	T	F
T	F	T
F	T	T
F	F	F

MATERIAL CONDITIONAL

One of the most common sentential connectives of English is 'if . . . then . . .'. A simple example is

> If Jones got the job then he applied for it.

This can be paraphrased as

> Either it is not the case that Jones got the job or Jones applied for the job

which can be symbolized as

$$\sim G \vee A$$

where 'G' abbreviates 'Jones got the job' and 'A' abbreviates 'Jones applied for the job'. It will be convenient to have a symbol in *SL* that expresses the truth-functional sense of 'if . . . then . . .'; we introduce '⊃' (horseshoe) for this purpose. The sentence 'If Jones got the job then Jones applied for the job' can then be symbolized as

G ⊃ A

A sentence of the form **P** ⊃ **Q**, where **P** and **Q** are sentences of *SL*, is a **material conditional. P**, the sentence on the left of the '⊃', is the **antecedent,** and **Q**, the sentence on the right of the '⊃', is the **consequent** of the conditional. It is important to remember that, whenever we write a sentence of the form **P** ⊃ **Q**, we could express it as ~ **P** ∨ **Q**. A sentence of the form ~ **P** ∨ **Q** is a disjunction, and a disjunction is false in only one case—when both disjuncts are false. Thus a sentence of the form ~ **P** ∨ **Q** is false when ~ **P** is false and **Q** is false, that is, when **P** is true and **Q** is false. This is also the only case in which a sentence of the form **P** ⊃ **Q** is false, that is, when the antecedent is true and the consequent is false. The characteristic truth-table is shown here:

P	Q	P ⊃ Q
T	T	T
T	F	F
F	T	T
F	F	T

Informally we can regard the 'if' clause of an English conditional as the antecedent of that conditional and the 'then' clause as the consequent. Here is an example of an English conditional converted to a truth-functional paraphrase that is symbolized by the material conditional:

If Michelle is in Paris then she is in France.

Expressed in a truth-functional paraphrase this becomes

<u>If</u> Michelle is in Paris <u>then</u> Michelle is in France.

The truth-functional paraphrase can be symbolized as a material conditional

P ⊃ F

Notice that the truth-functional paraphrase is false if Michelle is in Paris but is not in France—that is, if the antecedent is true and the consequent is false. But the truth-functional paraphrase is true under all other conditions. Thus, if Michelle is in Paris and in France, the paraphrase is true. If Michelle is not in Paris but is somewhere else in France, the paraphrase is true. If Michelle is not in Paris and not in France, the paraphrase is true.

However, the material conditional is not adequate as a complete treatment of conditional sentences in English. Material conditionals are truth-functional, but conditionals in English frequently convey information that exceeds a truth-functional analysis. For instance, 'if . . . then . . .' constructions sometimes have a causal force that is lost in a truth-functional paraphrase. Consider:

1. If this rod is made of metal then it will expand when heated.
2. If this rod is made of metal then it will contract when heated.

Each of these sentences can be used to make a causal claim, to assert a causal relation between the substance of which the rod in question is composed and the reaction of the rod to heat. But sentence 1 is in accord with the laws of nature, and sentence 2 is not. So, as used to make causal claims, sentence 1 is true and sentence 2 is false, even if it is false that the rod is made of metal.

Now suppose we paraphrase these two sentences as material conditionals:

1a. If this rod is made of metal then this rod will expand when heated.

2a. If this rod is made of metal then this rod will contract when heated.

These paraphrases can be symbolized as

1b. $M \supset E$

2b. $M \supset C$

where 'M' abbreviates 'The rod is made of metal', 'E' abbreviates 'This rod will expand when heated', and 'C' abbreviates 'This rod will contract when heated'. Remember that a material conditional is true if the antecedent is false. If the rod in the example is not made of metal, then both sentences 1a and 2a, and consequently their symbolizations 1b and 2b, are true. Sentence 1 says more than either 1a or 1b, and sentence 2 says more than either 2a or 2b. The fact that sentence 2 is false, whereas 2a and 2b are both true, shows this. It follows that when they are used to assert a causal relation, sentences 1 and 2, like many other English conditionals, are not truth-functional compounds. When it is and when it is not appropriate to paraphrase such sentences as material conditionals will be discussed further in Section 2.3.

Here are further examples of English sentences that can be paraphrased by using 'if . . . then . . .', but here and elsewhere we must keep in mind that sometimes information contained in the English conditionals will be lost in truth-functional paraphrasing.

Larry will become wealthy provided that he inherits the family fortune

can be paraphrased as

If Larry inherits the family fortune then Larry will become wealthy

which can be symbolized as

$F \supset W$

where 'F' abbreviates 'Larry inherits the family fortune' and 'W' abbreviates 'Larry will become wealthy'.

The Democratic candidate will win the election if he wins in the big cities

can be paraphrased as

If the Democratic candidate wins in the big cities then the Democratic candidate will win the election

which can be symbolized as '$C \supset E$', where 'C' abbreviates 'The Democratic candidate wins in the big cities' and 'E' abbreviates 'The Democratic candidate will win the election'.

Betty is in London only if Betty is in England

can be paraphrased as

If Betty is in London then Betty is in England

which can be symbolized as '$L \supset E$', where 'L' abbreviates 'Betty is in London' and 'E' abbreviates 'Betty is in England'. In this case be sure to notice the order in which the sentences are paraphrased. A common mistake in paraphrasing the sentential connective 'only if' is to ignore the word 'only' and reverse the order of the sentences. It is *incorrect* to paraphrase the original as 'If Betty is in England then Betty is in London'.

A connective that can be paraphrased either as a disjunction or as a conditional is 'unless'. Consider the sentence

This plant will die unless it is watered.

The only circumstance under which this sentence is false is the situation in which this plant does not die and is not watered. If either of the sentences that 'unless' connects is true, then the whole sentence is true. The simplest paraphrase is to treat the sentence as the disjunction

Either this plant will die or it is watered

which can be symbolized as

$D \vee W$

We can also understand the sentence 'This plant will die unless it is watered' as expressing a conditional:

If it is not the case that it is watered, then this plant will die which can be symbolized as

$$\sim W \supset D$$

Equally well, we can understand the sentence as expressing the equivalent conditional:

<u>If</u> <u>it is not the case that</u> this plant will die, <u>then</u> it is watered

which when symbolized is

$$\sim D \supset W$$

The two conditional paraphrases look different from each other and from the disjunction, but they make identical truth-functional claims. The disjunction claims that at least one of its component sentences is true. Each of the conditionals claims that, if one of two component sentences is not true, the other one is true. Here is a table that shows the truth-functional equivalence of the symbolizations for ‘unless’:

Truth Conditions for ‘Unless’

P	Q	P ∨ Q	~ P ⊃ Q	~ Q ⊃ P
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	F

MATERIAL BICONDITIONAL

In English the connective ‘if and only if’ is used to express more than either the connective ‘if’ or the connective ‘only if’. For example

John will get an A in the course if and only if he does well on the final examination

can be paraphrased as

<u>Both</u> (<u>if</u> John will get an A in the course <u>then</u> John does well on the final examination) <u>and</u> (<u>if</u> John does well on the final examination <u>then</u> John will get an A in the course).

We can symbolize the paraphrase as

$$(C \supset E)\ \&\ (E \supset C)$$

where 'C' abbreviates 'John will get an A in the course' and 'E' abbreviates 'John does well on the final examination'. The original sentence can also be paraphrased as

> Either (both John will get an A in the course and John does well on the final examination) or (both it is not the case that John will get an A in the course and it is not the case that John does well on the final examination).

Using the same abbreviations, this paraphrase is symbolized as

> (C & E) ∨ (~ C & ~ E)

Both of these paraphrases and their corresponding symbolizations are truth-functional compounds. Each is true just in case either both atomic sentences are true or both atomic sentences are false. We introduce the connective '≡' (triple bar) to capture the truth-functional use of the connective 'if and only if'. The original English sentence can be symbolized as

> C ≡ E

A sentence of the form

> **P ≡ Q**

where **P** and **Q** are sentences of *SL*, is a **material biconditional.** Informally we shall use the term "material biconditional" when describing English sentences that can be symbolized as material biconditionals in *SL*. Here is the characteristic truth-table for '≡':

P	**Q**	**P ≡ Q**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

The connective 'just in case' is sometimes used in English as an equivalent to 'if and only if'.

> Andy will win the lottery just in case Andy has the winning ticket

can be properly paraphrased as

> Andy will win the lottery if and only if Andy has the winning ticket

and symbolized as

> W ≡ T

However, care must be taken when paraphrasing 'just in case' because this connective sometimes is used in ways *not* equivalent to 'if and only if'. Consider

Marty takes her umbrella to work just in case it rains.

This does not mean 'Marty takes her umbrella to work if and only if it rains'. Rather, the sentence means

Marty takes her umbrella to work because it may rain.

SUMMARY OF SOME COMMON CONNECTIVES

Note that we use lowercase boldface '**p**' and '**q**' to designate sentences of English and uppercase boldface '**P**' and '**Q**' to designate sentences of *SL*.

English Connectives	*Paraphrases*	*Symbolizations*
not **p**	it is not the case that **p**	~ **P**
p and **q** **p** but **q** **p** however **q** **p** although **q** **p** nevertheless **q** **p** nonetheless **q** **p** moreover **q**	both **p** and **q**	**P** & **Q**
p or **q**	either **p** or **q**	**P** ∨ **Q**
p or **q** [*exclusive*]	both either **p** or **q** and it is not the case that both **p** and **q**	(**P** ∨ **Q**) & ~ (**P** & **Q**)
if **p** then **q** **p** only if **q** **q** if **p** **q** provided that **p** **q** given **p**	if **p** then **q**	**P** ⊃ **Q**
p if and only if **q** **p** if but only if **q** **p** just in case **q**	**p** if and only if **q**	**P** ≡ **Q**
neither **p** nor **q**	both it is not the case that **p** and it is not the case that **q** it is not the case that either **p** or **q**	~ **P** & ~ **Q** ~ (**P** ∨ **Q**)
not both **p** and **q**	it is not the case that both **p** and **q** either it is not the case that **p** or it is not the case that **q**	~ (**P** & **Q**) ~ **P** ∨ ~ **Q**
p unless **q**	either **p** or **q** if it is not the case that **p** then **q** if it is not the case that **q** then **p**	**P** ∨ **Q** ~ **P** ⊃ **Q** ~ **Q** ⊃ **P**

The connective 'because' is not truth-functional. ('Because' can join two true sentences resulting in a true sentence and 'because' can join two true sentences resulting in a false sentence.) Hence 'Marty takes her umbrella to work just in case it rains' should be symbolized by a single sentence letter such as 'M'.

In our discussion of the material conditional and the material biconditional, we have been careful to distinguish among connectives such as 'if', 'only if', and 'if and only if'. These distinctions are very important in logic, philosophy, and mathematics. However, in everyday discourse people speak casually. For example, people may use 'if' or 'only if' when they mean 'if and only if'. Our general policy in this book is to take disjunctions and conditionals in their weaker rather than their stronger senses. That is, normally 'or' will be read in the inclusive sense, and 'if . . . then . . .' (and other conditional connectives) will be taken in the material conditional sense (not the biconditional sense). When stronger readings are intended, we will indicate that by explicitly using expressions such as 'either . . . or . . . but not both' and 'if and only if'.

2.1E EXERCISES

1. For each of the following sentences, construct a truth-functional paraphrase and symbolize the paraphrase in *SL*. Use these abbreviations:

A: Albert jogs regularly.
B: Bob jogs regularly.
C: Carol jogs regularly.

a. Bob and Carol jog regularly.
*b. Bob does not jog regularly, but Carol does.
c. Either Bob jogs regularly or Carol jogs regularly.
*d. Albert jogs regularly and so does Carol.
e. Neither Bob nor Carol jogs regularly.
*f. Bob does jog regularly; however, Albert doesn't.
g. Bob doesn't jog regularly unless Carol jogs regularly.
*h. Albert and Bob and also Carol do not jog regularly.
i. Either Bob jogs regularly or Albert jogs regularly, but they don't both jog regularly.
*j. Although Carol doesn't jog regularly, either Bob or Albert does.
k. It is not the case that Carol or Bob jogs regularly; moreover Albert doesn't jog regularly either.
*l. It is not the case that Albert, Bob, or Carol jogs regularly.
m. Either Albert jogs regularly or he doesn't.
*n. Neither Albert nor Carol nor Bob jogs regularly.

2. Using the abbreviations given in Exercise 1, construct idiomatic English sentences from the following sentences of *SL*:

a. A & B
*b. $A \vee \sim A$

c. A ∨ C
*d. ~ (A ∨ C)
e. ~ A & ~ C
*f. ~ ~ B
g. B & (A ∨ C)
*h. (A ∨ C) & ~ (A & C)
i. (A & C) & B
*j. ~ A ∨ (~ B ∨ ~ C)
k. (B ∨ C) ∨ ~ (B ∨ C)

3. Assuming that 'Albert jogs regularly' is true, 'Bob jogs regularly' is false, and 'Carol jogs regularly' is true, which of the symbolic sentences in Exercise 2 are true and which are false? Use your knowledge of the characteristic truth-tables in answering.

4. Paraphrase each of the following using the phrase 'it is not the case that'. Symbolize the results, indicating what your abbreviations are.

a. Some joggers are not marathon runners.
*b. Bob is not a marathon runner.
c. Each and every marathon runner is not lazy.
*d. Some joggers are unhealthy.
e. Nobody is perfect.

5. For each of the following sentences, construct a truth-functional paraphrase and symbolize the paraphrase in *SL*. Use these abbreviations:

A: Albert jogs regularly.
B: Bob jogs regularly.
C: Carol jogs regularly.
L: Bob is lazy.
M: Carol is a marathon runner.
H: Albert is healthy.

a. If Bob jogs regularly he is not lazy.
*b. If Bob is not lazy he jogs regularly.
c. Bob jogs regularly if and only if he is not lazy.
*d. Carol is a marathon runner only if she jogs regularly.
e. Carol is a marathon runner if and only if she jogs regularly.
*f. If Carol jogs regularly, then if Bob is not lazy he jogs regularly.
g. If both Carol and Bob jog regularly, then Albert does too.
*h. If either Carol or Bob jogs regularly, then Albert does too.
i. If either Carol or Bob does not jog regularly, then Albert doesn't either.
*j. If neither Carol nor Bob jogs regularly, then Albert doesn't either.
k. If Albert is healthy and Bob is not lazy, then both jog regularly.
*l. If Albert is healthy, he jogs regularly if and only if Bob does.
m. Assuming Carol is not a marathon runner, she jogs regularly if and only if Albert and Bob both jog regularly.
*n. Although Albert is healthy he does not jog regularly, but Carol does jog regularly if Bob does.
o. If Carol is a marathon runner and Bob is not lazy and Albert is healthy, then they all jog regularly.

*p. If Albert jogs regularly, then Carol does provided Bob does.
q. If Albert jogs regularly if Carol does, then Albert is healthy and Carol is a marathon runner.
*r. If Albert is healthy if he jogs regularly, then if Bob is lazy he doesn't jog regularly.
s. If Albert jogs regularly if either Carol or Bob does, then Albert is healthy and Bob isn't lazy.
*t. If Albert is not healthy, then Bob and Albert do not both jog regularly.

6. Using the abbreviations given in Exercise 5, construct idiomatic English sentences from the following sentences of *SL*.

a. $L \vee \sim L$
*b. $M \supset C$
c. $A \equiv H$
*d. $C \,\&\, \sim B$
e. $\sim B \,\&\, \sim C$
*f. $[A \vee (B \vee C)] \supset [A \,\&\, (B \,\&\, C)]$
g. $(\sim A \vee \sim C) \supset B$
*h. $\sim (A \vee C) \supset B$
i. $C \supset (A \,\&\, \sim B)$
*j. $B \equiv (\sim L \,\&\, A)$
k. $C \,\&\, \sim C$
*l. $A \,\&\, (C \equiv B)$
m. $(L \supset L) \,\&\, B$
*n. $\sim \sim H \,\&\, \sim A$
o. $\sim A \supset (\sim B \supset \sim C)$
*p. $(C \supset A) \,\&\, (A \supset B)$
q. $\sim A \,\&\, (B \equiv \sim L)$
*r. $(H \supset A) \supset (\sim L \supset B)$

7. Give a truth-functional paraphrase for each of the following, and symbolize the paraphrase in *SL*.

a. Neither men nor women are from Mars or Venus.
*b. This dog won't hunt; moreover he is not even a good pet.
c. Not both Butch Cassidy and the Sundance Kid escaped.
*d. The tea will not taste robust unless it steeps for a while.
e. That lady was both cut in half and torn asunder unless it was a magic trick.
*f. Neither wind nor rain nor dark of night will stop the mail.
g. The prisoner will receive either a life sentence or the death penalty.
*h. Unless snowstorms arrive, skiing and snowboarding will be impossible.

8. What are the truth-conditions for the exclusive 'or'? How might the exclusive 'or' be expressed as a biconditional?