T. Parent

DECISION THEORY PRIMER

**If you're in a hurry, you should at least take a look at the last page of this handout.*

Decisions under Risk

Draw once from a deck of 52 cards

ACTS \ STATES	*Win (State1)*	*Lose (State2)*
Bet Red Card	+1M₸ — 50%	–1M₸ — 50%
Bet Face Card	+3M₸ — 23%	–1M₸ — 77%

Expected payout ("EP") of an Act (where the states are mutually exclusive and exhaustive):

EP = [$\text{Payout}_{\text{State1}}$ x *Prob*(State1)] + [$\text{Payout}_{\text{State2}}$ x *Prob*(State2)]

EP of Betting Red:
(1M₸ x .5) + (–1M₸ x .5) = 500K+ –500K₸ = 0₸

EP of Betting Face Card:
(3M₸ x .23) + (–1M₸ x .77) = 690K₸ + –770K₸ = –80₸

Assumption: Financial gains and losses are the only things that matter. (?!?) Given *Assumption*, you should do what has the greatest EP.

Hence, you should bet red.

Decisions under Ignorance

ACTS \ STATES	*You get job*	*You don't get job*
Lawyer	+1M ₸ — [unknown]	–5M ₸ — [unknown]
Philosophy Prof.	+1M₸ — [unknown]	–1M ₸ — [unknown]

Dominance Rule: Do not choose dominated acts.
A dominates *B* iff the outcome of doing *A* is always at least as good, and sometimes better, than the outcome of doing *B*.

Hence, you should be a philosophy professor.

But suppose the decision under ignorance is more complicated…

	You get job	*You don't get job*
Philosophy Prof.	+1M ₸ [unknown]	–1M ₸ [unknown]
Engineer	+3M₸ [unknown]	–2M₸ [unknown]
M.D.	+6M₸ [unknown]	–5M₸ [unknown]

Maximin Rule: Choose the act with the largest minimum outcome. ("Play It Safe")

Hence, you should be a philosophy professor.
(What if there is a tie? "Lexical" Maximin Rule: Do what has the largest "second worst" payout. If there's still a tie, do what has the largest "third worst" payout. And so on.)

Minimax Regret Rule: Choose the act with the smallest regret, i.e., the smallest opportunity loss. ("Regret Avoidance") [This is not just the "Maximax Rule," which would advises doing what has the greatest possible maximum payout.]

Regret of an Act *A* given a State *S*:
$R_{A/S}$ = Maximum possible payoff given *S* – $\text{Payout}_{A/S}$

<u>Regret Table for the previous decision:</u>

	You get job	*You don't get job*
Philosophy Prof.	–5M₸ [unknown]	0 ₸ [unknown]
Engineer	–3M₸ [unknown]	–1M₸ [unknown]
M.D.	0₸ [unknown]	–4M₸ [unknown]

The maximum regret possible for a philosophy professor is –5M₸. The maximum regret possible for an engineer is –3M₸. The maximum regret possible for an M.D. is –4M₸. The Rule says you should choose what has the smallest maximum regret possible.

Hence, you should be an engineer. [The "Maximax" rule would have you be an M.D.]

Optimism-Pessimism Rule: Gauge your level of optimism by picking a number *n* between 0 and 1, where a bigger number means higher optimism. ("Weight Choices by Optimism Level")

OP-Payout of an Act *A*:
(Greatest possible payout for *A* x *n*) + (Least possible payout for *A* x (1–*n*))

Suppose your level of optimism is .7. Then:

OP-Payout of being a Phil Prof:
(1M₸ x .7) + (–1M₸ x .3) = 700K+ –300K₸ = 400K₸

OP-Payout of being an Engineer:
(3M₸ x .7) + (–2M₸ x .3) = 2.1M₸ + –600K₸ = 1.5M₸

OP-Payout of being an M.D.
(6M₸ x .7) + (–5M₸ x .3) = 4.2M₸ + –1.5M₸ = 2.7M₸

Hence, you should be an M.D.

Whereas, suppose your level of optimism is .3. Then:

OP-Payout of being a Phil Prof:
(1M₸ x .3) + (–1M₸ x .7) = 300K+ –700K₸ = –400K₸

OP-Payout of being an Engineer:
(3M₸ x .3) + (–2M₸ x .7) = 900K₸ + –1.4M₸ = –500K₸

OP-Payout of being an M.D.
(6M₸ x .3) + (–5M₸ x .7) = 1.8M₸ + –4.5M₸ = –2.7M₸

Hence, you should be a philosophy professor.

Principle of Insufficient Reason: In your ignorance, assume the states are equally probable and treat it like a Decision under Risk. ("Ignorance means Equiprobable")

"EP" of being a Phil Prof:
(1M₸ x .5) + (–1M₸ x .5) = 500K+ –500K₸ = 0₸

"EP" of being an Engineer:
(3M₸ x .5) + (–2M₸ x .5) = 1.5M₸ + –1M₸ = 500K₸

"EP" of being an M.D.
(6M₸ x .5) + (–5M₸ x .5) = 3M₸ + –2.5M₸ = 500K₸

Hence, you should be either an Engineer or an M.D. (Doesn't matter which.)

Retrospective on Decision Theory

The decisions we've just reviewed are quite unusual in several ways:

> -They assume financial gains and losses are they only things that matter;
> -They assume you can identify the gains and losses for every act/state pairing;
> -They assume that you can identify the probabilities precisely (Decisions under Risk) or not at all (Decisions under Ignorance)

Real life decisions usually do not fit any of these assumptions. Still, Decision Theory at least provides some *clarity* on what it means to be a rational decision maker.

Moreover, thinking about choices in relation to Decision Theory is a way of performing some crucial "checks" on your decision-making process.

First, if you make a list of "pros and cons" under each of your options, you are already doing much better than most people. In Decision Theory, this basically happens when we:

1. **Identify the *possible outcomes* for each act under consideration.**

But more than that, Decision Theory forces us to:

2. **Consider the *probability* of each outcome for each act.**

So if we have a list of pros and cons, it might be good to consider *how likely* each of the positives and negatives are, under each option. The likelihood of the various positives and negatives may very well affect how we view those options. For each act, is the positive outcome a sure thing? Or is it more of a pipe dream?

The point is ultimately to discern what action is the best bet, considering all the possible outcomes and their probabilities. This happens in Decision Theory when we:

3. ***Compare* the probability-outcome pairs for each act.**

Such a comparison guides us to which act is most likely to yield a good outcome and/or divert a bad outcome.

In sum, identifying outcomes, considering probabilities, and comparing the results is a good procedure to follow in your decision making, regardless of whether you use Decision Theory. It discourages you from deciding too hastily, i.e., deciding without adequately considering the crucial aspects of the decision.