

Tarski, “The Semantic Conception of Truth...”

Two Adequacy Conditions on a Theory of Truth

Materially adequate: It should be a correspondence theory of truth. More precisely (?), it should entail all instances of the following T-schema:

(T) X is true iff *p*.

...where ‘*p*’ is replaced with a sentence, and ‘X’ is replaced with a name for that sentence (typically, a quote-name).

Formally correct: It must “specify the words or concepts which we wish to use in defining the notion of truth; and we must also give the formal rules to which the definition should conform” (p. 86)

*One more: We “*apply the term ‘true’ to sentences*” vs. mysterious “propositions” (*ibid.*)

Truth as a Semantic notion:

“Semantics is a discipline which...deals with certain relations between expressions of a language and the objects (or ‘states of affairs’) ‘referred to’” (pp. 87-8).

“it turns out that the simplest and most natural way of obtaining an exact definition of truth is one which involves the use of other semantic notions, e.g., the notion of satisfaction...[Thus] the problem of defining truth proves to be closely related to the more general problem of setting up the foundations of theoretical semantics” (p. 88).

X

Swipe at Positivism: Semantics “has no pretensions...as a device for establishing that everyone except the speaker and his friends is speaking nonsense” (*ibid.*)

Specifying a Language’s Structure:

Define the Lexicon: “we must indicate all words which we decide to use without defining them, and which are called ‘*undefined* (or *primitive*) *terms*’; and we must give the so-called *rules of definition* for introducing new or *defined terms*” (*ibid.*)

Define the Well-Formed (i.e. Grammatical) Sentences: “we must set up criteria for distinguishing within the class of expressions those which we call ‘sentences’” (*ibid.*)

Specify the Axioms: “we must indicate all axioms (or *primitive sentences*), i.e., those sentences which we decide to assert without proof; and we must give the so-called *rules of inference* (or *rules of proof*) by means of which we can deduce new asserted sentences from the other sentences which have been previously asserted” (pp. 88-9).

All this is to be done *formally* (though a language with a specifiable structure is not always formally specifiable. Such a language might be the language of an ideal science).

NOTE: “the only languages with a specified structure are the formalized languages of various systems of deductive logic” Therefore: truth can be defined “*in a rigorous way only for those languages whose structure has been exactly specified*” (p. 89)

Still, we can “approximately” define truth in a natural language by “replacing a natural language (or a portion of it in which we are interested) by one whose structure is exactly specified, and which diverges from the given language ‘as little as possible’” (*ibid.*)

But even formalized languages face difficulties in being exactly specified...

The Antinomy of the Liar

Intuitively, any sentence is either true or false (but not both). (This is the Law of Excluded Middle.) Yet LEM will be violated if (L) is a sentence of the language...

(L): (L) is not true.

Is (L) true or false? In fact, we can show that it is true iff it is false—meaning it is either *both* true and false or *neither* true nor false. Either way, (L) violates LEM.

Assumptions behind the Liar

(I) *The Language is Semantically Closed*: The language contains names for its own expressions, and semantic terms like ‘true’ that are defined on its own expressions.

(II) *Classical Logic*: the laws of “ordinary logic” hold, including LEM.

(III) *Assertability of (L*)*: The reasoning that shows LEM is violated assumes that the following is an axiom or theorem:

(L*) ‘(L) is not true’ = (L).

Tarski claims (III) is not essential to the paradox, and views (II) as non-negotiable. Thus, “we decide *not to use any language which is semantically closed*” (p. 90). (Still, if natural language allows the Liar, does that mean that our ordinary notion of ‘true’ is inconsistent? Tarski: Nat’l language doesn’t have a specifiable structure.)

OL and ML:

Since the language cannot contain the term ‘true’ (defined on its own sentences), we must talk about what’s “true” in the language using a *different* language (the “metalanguage” vs. the “object language”). N.B., the distinction between ML and OL is relative...so in principle there is an infinite hierarchy of languages.

Since our theory in ML must entail all instances of (T), and since these use an OL-sentence on the right-hand side, ML must contain every OL expression. (Relatedly, ML also must contain names for every OL-expression.) Tarski notes, however, that ML need not contain OL if ML contains a *translation* of every OL expression.

The project is thus to define in ML the predicate ‘true’ when (and only when!) it applies to OL-sentences...where this entails every instance of the (T)-schema for OL. Since only ML contains semantic terms for OL-expressions, ML is “essentially richer” than OL.

Defining ‘True’ on OL-sentences

Tarski defines ‘true’ using the notion of satisfaction, where “satisfaction” is a relation between sentential functions and objects. “Sentential functions” are defined recursively:

- We first define the primitive sentential functions, which are the non-compound predicates and atomic sentences.

- Then, we give the rules for constructing the compound sentential functions from the non-compound ones. These rules are merely the rules that say how to join sentential functions together with truth-functional connectives like ‘and’, ‘or’, ‘not’, etc. Also, we give the rules for adding quantifiers.

(Sentence are then defined as sentential functions with no free variables.)

Satisfaction *could* be defined as the relation which holds between a sentential function and some object(s) iff the sentential function outputs “true” when the input is the object(s). But that would define satisfaction by the predicate ‘true’, and we want to avoid circularity given that our aim is to define ‘true’ by the notion of satisfaction.

Thus Tarski instead goes for a recursive definition of satisfaction, which ends up being best captured as a relation between sentential functions and *sequences* of objects $\langle o_1 \dots o_n \rangle$. (We use sequences rather than individual objects, so we can always talk about the same kind of thing [sequences of length n] as satisfying a predicate with n -places.)

As a preliminary to the recursive definition, we first give a list which maps finitely many names to their corresponding referents. E.g., suppose our language has two names, ‘Snoopy’ and ‘Charlie Brown.’ Then we will first give the following axioms:

1. ‘Snoopy’ refers to Snoopy.
2. ‘Charlie Brown’ refers to Charlie Brown.

We now list the satisfaction-conditions for finitely many non-compound predicates. Suppose our language contains, in addition to the two names, the predicates ‘ x is a dog’ and ‘ x is owned by y ’. Then we would add the following two axioms:

3. ‘ x is a dog’ is satisfied by a sequence σ iff the first member of σ is a dog.
4. ‘ x is owned by y ’ is satisfied by a sequence σ iff the first member of σ is owned by the second member of σ .

Axioms like 1-4 are standardly referred to as the “base clauses” of the Tarskian theory. We now give the “inductive rules” which specify the sequences satisfying compound predicates, by appeal to what satisfies their component parts. In general we can say that, where σ is any sequence:

5. σ satisfies “ $\sim\Phi$ ” iff σ does not satisfy Φ .
6. σ satisfies “ $\Phi \ \& \ \Psi$ ” iff σ satisfies Φ and σ satisfies Ψ .
7. σ satisfies “ $\Phi \vee \Psi$ ” iff σ satisfies Φ or σ satisfies Ψ .
8. σ satisfies “ $\Phi \supset \Psi$ ” iff σ satisfies Φ only if σ satisfies Ψ .
9. σ satisfies “ $\Phi \equiv \Psi$ ” iff σ satisfies Φ if and only if σ satisfies Ψ .

In addition, we specify when a sequence satisfies a sentence:

10. Where $\langle \alpha_1 \dots \alpha_n \rangle$ is a sequence of names and Φ is an n -place predicate, *any* sequence σ satisfies " $\Phi(\alpha_1 \dots \alpha_n)$ " iff Φ is satisfied by *some* sequence such that α_1 refers to its first member, and α_2 refers to its second member, and...and α_n refers to its n th member.
11. Any σ satisfies " $\exists x \Phi$ " iff some sequence satisfies Φ (with at most x free).
12. Any σ satisfies " $\forall x \Phi$ " iff every sequence satisfies Φ (with at most x free).

Given how Tarski defines satisfaction on sentences, it turns out that a sentence is either satisfied by *all* sequences or none. And the sentences which are satisfied by all sequences are precisely the true ones. So truth is defined thus:

(Def) A sentence is true iff it is satisfied by all sequences.

The T-biconditionals then can be derived from (Def) and clauses like 1-12 (assuming the classical rules of inference have been specified for the language), since the satisfaction-conditions in the base clauses are specified explicitly. Here's how one derivation goes:

13. ‘Snoopy is a dog’ is true iff ‘Snoopy is a dog’ is satisfied by any sequence σ .
[From (Def)]
14. Any σ satisfies ‘Snoopy is a dog’ iff ‘ x is a dog’ is satisfied by some sequence such that ‘Snoopy’ refers to its first member.
[From 10]
15. Any σ satisfies ‘Snoopy is a dog’ iff some sequence is such that the first member is a dog and is Snoopy.
[From 1, 3, 14]
16. Any σ satisfies ‘Snoopy is a dog’ iff Snoopy is a dog.
[From 15]
- (T1) ‘Snoopy is a dog’ is true iff Snoopy is a dog.
[From 13, 16]

Remarks on the Definition:

Laws of classical logic can be derived from the theory (distinguish these laws from rules of inference).

Incompleteness. “This sentence is not provable.”

Other semantic notions are definable in terms of satisfaction and will be subject to a similar treatment, to avoid paradoxes.

Objections and Replies

Preface: The theory is not the “only right” one. “I do not understand what is at stake in such disputes; for the problem itself is so vague that no definite solution is possible... [The dispute is] based upon the belief that every word has only one ‘real’ meaning (a kind of Platonic or Aristotelian idea)...We should reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by one word; we should try to make these concepts as clear as possible...we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved” (p. 94).

The Circularity Objection

Truth-functional connectives are used in formulating the definition of satisfaction. Tarski: This would damn any theory of truth. Also, if we're only defining 'true' on OL-sentences, it doesn't matter if we use 'true' in ML, where we articulate the definition.

Brevity Objection

Shouldn't the (T)-schema be replaced with "X is true if, and only if, p is true." Tarski: No, for then you would need to replace ' p ' with a sentence-name rather than a sentence.

Redundancy Objection:

Given the (T)-sentences, isn't the predicate 'true' eliminable without loss? Tarski: We need 'true' to say "All consequences of true sentences are true"...also "The first sentence written by Plato is true" (when that sentence is unknown). Still, we could eliminate all occurrences of 'true' using the notion of satisfaction, given Tarski's definition.

Relevance Worry

Is Tarski's notion of "true" truly a regimented version of Aristotle's? Or of the everyday use of 'true'? Tarski: It captures the "intuitive content" of Aristotle's notion, and it "conform[s] to a very considerable extend with the common-sense usage." Poll results.

Anti-Realist Objection

"logic finds itself involved in a most uncritical realism" Tarski: The definition is compatible with critical realism, idealism, etc. And you wouldn't want a theory which implies that possibly 'snow is white' is true even if it's not the case that snow is white.

Does Semantics Assume any Metaphysics?

Tarski: Ontology "has hardly any connections with semantics." Semantics is part of deductive science, and doesn't use specialized metaphysical terms. You could introduce metaphysical axioms, but you could do that for an empirical theory as well.

What Use is Semantics?

Defining a notion precisely helps anyone who uses the notion, and semantic notions are present in other disciplines (literature, history, psychology, linguistics, scientific methodology, metamathematics.) Also: "it is inimical to the progress of science to measure the importance of any research exclusively or chiefly in terms of its usefulness" (p. 103).

Davidson, “Truth and Meaning”

The Name Theory of Meaning: Every term is a name for an object, property or relation.
Problem: Bradley’s Regress. Suppose ‘Theaeteus’ names Theatetus, and ‘flies’ names the property of flying. How do you get the sentential-meaning of “Theatetus flies” from the meanings of ‘Theatetus’ and ‘flies’? Answer: Concatenation signals that the subject “participates in” the property denoted by ‘flies’. Objection: Vacuous—The problem is precisely to say how this sentential-meaning comes about.

Second problem: ‘the father of Annette’: Concatenating ‘the father of’ and ‘Annette’ yields a term for the father of Annette. Yet this doesn’t identify a *denotation* for ‘the father of’. (Circularity doesn’t matter, since the issue is to show how composed meanings arise from primitive ones, not to account for the primitive meanings themselves.)

But: We could view ‘the father of’ not as denoting an entity, but as having a systematic effect on the meaning of other terms. Just have your theory say the following:

- (1) ‘Annette’ refers to Annette.
- (2) A phrase of the form “the father of *t*” denotes the father of what *t* refers to.

Frege’s Theory: ‘the father of *t*’ is a function mapping persons to their fathers.

Problems: The Slingshot argument (See Separate Handout). The Bradley Regress again.

Sentences as names for Meanings: Let syntax determine which strings are sentences; let a dictionary give the meanings of sub-sentential parts. From this, derive sentences of the form “*s* means *m*,” where *s* is a sentence (formally described) and *m* is its meaning.

Problems: Belief-ascriptions are non-compositional. The Bradley Regress once again.

Sentences as names for states-of-affairs: Let syntax and the dictionary be as before. Derive sentences of the form “*s* means *p*,” where ‘*p*’ is replaced with the sentence itself.
Problem. Sentences of the form “*s* means *p*” are non-extensional.

Davidson’s Theory: Let syntax and the dictionary be as before. Derive sentences of the form “*s* is T iff *p*,” where ‘T’ is a predicate that makes these biconditionals true whenever ‘*s*’ is replaced with the quote-name for the sentence replacing ‘*p*’. ‘True’ is such a predicate; so, *Tarski’s theory of truth is an adequate theory of sentential meaning*.

“a theory of meaning for a language L shows ‘how the meanings of sentences depend upon the meaning of words’ if it contains a (recursive) definition of truth-in-L” (p. 117)
“the definition works by giving necessary and sufficient conditions for the truth of every sentence, and to give truth conditions is a way of giving the meaning” (p. 118)

Davidson’s Theory, Pros and Cons

The Theory is Empirical. Each instance of the (T)-schema provides a testable hypothesis, e.g., the (syntactically described) string ‘Snow is white’ is true if and only if snow (the stuff) is white.

Objection: Aren't the T-sentences uninformative? Davidson's Reply: "The theory reveals nothing new about the conditions under which an individual sentence is true; it does not make those conditions any clearer than the sentence itself does. The work of the theory is in relating the known truth conditions of each sentence to those aspects ('words') of the sentence that recur in other sentences" (p. 118) Also: When ML does not contain OL, the problem disappears.

Testing a Davidsonian theory in practice: See pp. 119, second half of the first column.

Can you apply Tarskian formalism to Natural Language? Tarski says 'no' due to (a) the semantic paradoxes, and (b) the confused and amorphous features of natural language.

Reply to (a): "it is not really clear how unfair to Urdu or to Hindi it would be to view the range of their quantifiers as insufficient to yield an explicit definition of 'true-in-Urdu' or 'true-in-Hindi'...there may...always be something we grasp in understanding the language of another (the concept of truth) that we cannot communicate" (p. 120)

Reply to (b): "Much of what is called for is just to mechanize as far as possible what we now do by art when we put ordinary English into one or another canonical notation. The point is not that the canonical notation is better than the rough original idiom, but rather that...we have as good a theory for the idiom as for its kept companion" (ibid.)

On Belief-Attributions: "all *these* problems will be translated without loss or gain into the metalanguage. But the central problem of...'believes that' will remain" (p. 121). Still, a Tarskian theory can say these have the form of a three-place- predicate, defined on ordered triples of <person, sentence, language>, It doesn't analyze 'believes that' in a way that explains its intensionality, but it is a virtue of Tarskian theory that it keeps separate the theory of logical form for *sentences* vs. the analysis of individual *words*.

Evaluative predicates:, a Tarskian theory treats sentences in the same way (by giving truth-conditions) whether or not they use evaluative vs. descriptive predicates. Besides, lots of compound predicates are non-compositional in meaning, e.g., 'is a good actress', and incorporating these predicates as primitives weakens the power of the theory.

Analytic vs. Synthetic "A truth definition does not distinguish between analytic sentences and others" (p. 122) But: logical truths have a formal characterization.

Demonstratives: Their containing sentences can be true in some contexts, but not others. "These complaints can be met, I think, though only by a fairly far-reaching revision in the theory of truth" (ibid.) Define 'true' relative to place, time, speaker, audience, etc. Although, this complicates the empirical testing of a translation.

Remaining Difficulties. Counterfactuals, sentences about probabilities, about causal relations, the logical role of adverbs, mass terms, sentences about belief/intention, action-verbs that imply purpose, and of course, imperatives and interrogatives.