

## Sufficient versus Necessary Conditions

Briefly: The sentence “If A, then B” says that A is a *sufficient condition* for B. Equivalently, it says that B is a *necessary condition* on A. (The order of A and B switches in the latter case.)

To illustrate further, let’s consider an example:

(1) *If* we clean our rooms, *then* Dad makes cookies.

In sentence (1), we say that cleaning our rooms is a *\*sufficient\** condition for Dad making cookies. I.e., for Dad to make cookies, it is *\*enough\** that we clean our rooms.

At the same time, (1) says that Dad making cookies is a *\*necessary\** condition of cleaning our rooms. I.e., Dad making cookies is a necessary consequence of our room-cleaning.

Now, it turns out that (1) says the same thing (logically speaking) as the following sentence:

(2) We clean our rooms *only if* Dad makes cookies.

Granted, the two sentences may have a different tone or different connotations. Yet both (1) and (2) tell us that, if ever we clean our rooms, you’ll find that Dad is making cookies. So for the purposes of logic, the two English sentences are saying the same thing: Cleaning our rooms is sufficient for Dad making cookies; equivalently, Dad making cookies is a necessary consequence of cleaning our rooms.

But contrast (1) and (2) with these sentences:

(3) *If* Dad makes cookies, *then* we clean our rooms.

(4) Dad makes cookies *only if* we clean our rooms.

In sentences like (3) and (4), things go the other way around: We are now saying that Dad making cookies is *\*sufficient\** for cleaning our rooms; in other words, cleaning our rooms is a *\*necessary\** result of Dad making cookies. Both (3) and (4) indicate that all Dad has to do is put cookies in the oven, and it’s a guarantee that we’re cleaning up our rooms

Notice that, since we can vary the grammatical order of English, (3) and (4) are equivalent to:

(5) We clean our rooms *if* Dad makes cookies.

This may seem confusing but notice that this is basically the same sentence as (3), except we placed the “iffy” part last (and deleted the word ‘then’). Similarly, the following is basically the same as (1) with the “iffy” part last:

(6) Dad makes us cookies *if* we clean our room.

So (6) is equivalent to (1) and to (2).

On the other hand, take heed that (6) is NOT equivalent to (4). That is so, even though the only difference is the word 'only'! The little word 'only' makes the difference between Dad making cookies being a sufficient versus a necessary condition on cleaning our rooms. (For the same reason, (5) is NOT equivalent to (2).)

Ok. Finally, consider that we can join (1) and (3) into one sentence:

(7) **If** we clean our rooms, **then** Dad makes cookies—and **if** Dad makes cookies, **then** we clean our rooms.

Observe that (7) is logically equivalent to (8) and to (9):

(8) Dad makes cookies **if but only if** we clean our rooms.

(9) **If** we clean our rooms, **then and only then** does Dad make cookies.

In (7)-(9), we are saying that Dad's making cookies is *\*both\** a sufficient *\*and\** a necessary condition for cleaning our rooms.

Why does this matter? As mentioned, some of the formal fallacies are committed because they are easily confused with some legitimate forms of reasoning. In particular, affirming the consequent is often conflated with *modus ponens*, whereas denying the antecedent is often conflated with *modus tollens*. Both confusions, however, have something in common. They wrongly assimilate a sentence of the form "If A, then B" with its converse "If B, then A." Equivalently, they confuse saying that A is *sufficient* for B with A is *necessary* for B. So to avoid formal fallacies, it is best to be clear on the difference between sufficient versus necessary conditions.