

Necessary and Sufficient Conditions

Let's start with an ordinary English sentence:

(1) Dad makes cookies only if we clean our rooms.

We might symbolize this sentence as

(2) $C \supset R$

Now, (1) says the exact same thing (logically speaking) as the following sentence:

(3) If Dad makes cookies, then we've cleaned our rooms.

Both (1) and (3) tell us that, whenever you find Dad making cookies, you'll find that we've cleaned our rooms. So for the purposes of logic, the two English sentences are stating the same thing. Consequently, (3) is symbolized the same as (1). Namely, it is symbolized by the sentence in (2).

In sentences like (1) and (3), we say that cleaning our rooms is a **necessary** condition on Dad's making cookies. I.e., in order for Dad to make us cookies, it is necessary that we clean our rooms.

Contrast (1) and (3) now with these sentences:

(4) Dad makes cookies if we clean our rooms.

(5) If we clean our rooms, Dad makes cookies.

Both of these sentences can be symbolized as:

(6) $R \supset C$

In sentences like (4) and (5), we say that our cleaning our rooms is a **sufficient condition**, i.e. it is **enough**, for Dad to make us cookies. All we have to do is clean up, and it's a guarantee that Dad's got cookies in the oven.

Now this is where it gets a little weird. It turns out that, for **logical** purposes, (7) is saying the same thing as (4) and (5).

(7) We clean our rooms only if Dad makes cookies.

This seems weird since in (7), it sounds like we're making some kind of bargain with Dad, whereas in (4) and (5), there is no bargaining tone. In (4) and (5): We're just saying that, if it turns out that we clean our rooms, then lo and behold, it will also turn out that Dad will have made cookies.

The differences here in tone, however, are irrelevant for the purposes of logic. What is relevant is that in (7), as well as in (4) and (5), we are making the same kind of conditional statement. We're saying in all these statements that WHENEVER you see us cleaning our rooms, you'll also find that Dad is making cookies. There may or may not be some kind of bargaining going on between Dad and ourselves, but this is irrelevant. What is relevant is that (4), (5), and (7) all say that whenever we've cleaned our rooms, it is also the case that Dad has made cookies. More technically, whenever the simple sentence "we've cleaned our rooms" is true, then the simple sentence "Dad makes cookies" is also true. That's what's important here, and that's what (4), (5), and (7) all have in common.

Ok. Finally, consider that we can conjoin conditional statements to get:

(8) If Dad makes cookies, then we've cleaned our rooms--and if we've cleaned our rooms, Dad makes cookies.

In symbols:

(9) $(C \supset R) \ \& \ (R \supset C)$

Note that (8) is logically equivalent to (10):

(10) Dad makes cookies if but only if we've cleaned our rooms.

Accordingly, the symbolization for (10) is logically equivalent to the symbolization for (8):

(11) $C \equiv R$

In these cases with the biconditional, we say that Dad's making cookies is a necessary *and* sufficient condition for us to clean our rooms.